Multiplexed Multispectral Filter Array by 3D Sphere Packing Design

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Introduction

Spectral imaging is a technique used in many fields to acquire spatio-spectral information. One method of acquisition is the snapshot with a Multispectral Filter Array (MSFA), but the signal-to-noise ratio (SNR) decreases with a higher number of bands. However, multiplexing (MUX) the bands to extend the design of the MSFA towards a Multiplexed Multispectral Filter Array (MMSFA) can improve the measurement SNR.

Method

Optimal Sphere Packing (OSP) MSFA is based on packing equal spheres in a cubic container, optimizing the density of spheres. The distance between a set of V spheres is given by

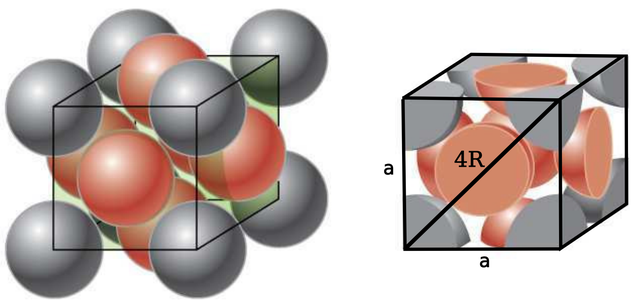
$$d^*(V) = \max\Big(\min_{1\le k_1<k_2\le V} D_{k_1,k_2}\Big), \qquad (1)$$

where D_{k_1,k_2} is the euclidean distance between the centers of all spheres, the theoretical upper bound SP density for the optimal MSFA as

$$\rho^*(V) = 2\sqrt[3]{\frac{(\sqrt{V}+1)^3}{4V\sqrt{2}}}, \qquad (2)$$

being the OSP density is $\rho_{\Lambda_3} = \frac{\pi}{\sqrt{18}} \approx 0.74$ [1].

- **Theorem:** *No packing of congruent balls in Euclidean three space has a density greater than that of the face-centered cubic packing ρ_{Λ_3} [2].*



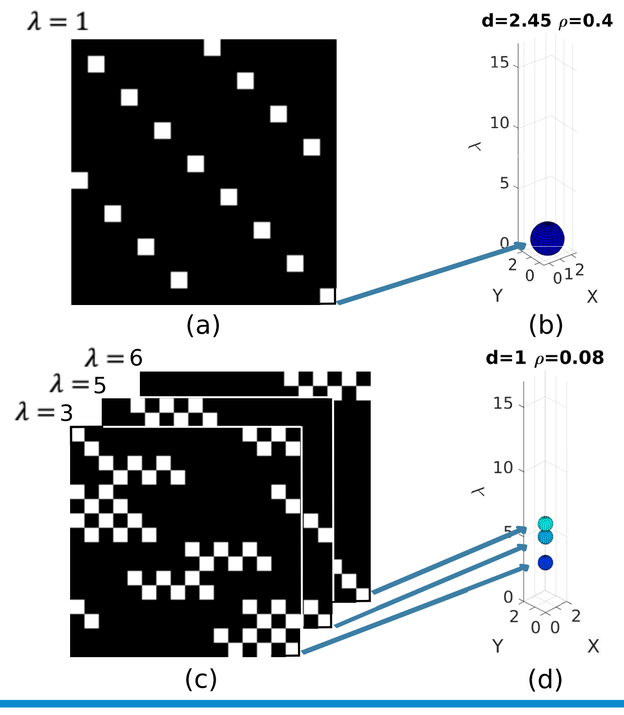
Face centered cubic unit cell

The acquisition of the spectral mosaic image projection of L spectral bands is

$$\mathbf{Y} = \sum_{l=1}^{L} \mathbf{X}_l \odot \mathbf{C}_l + \mathbf{\Omega}, \qquad (3)$$

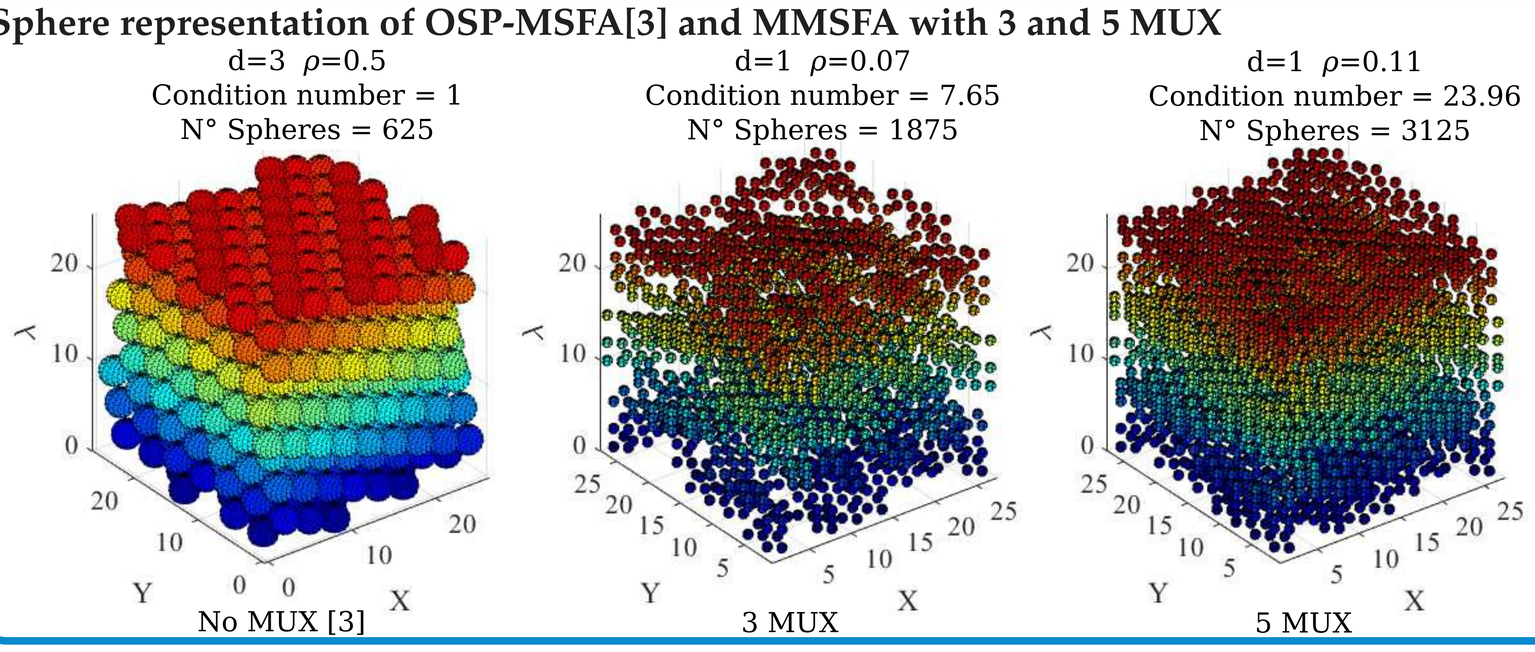
where $\mathbf{X}_l \in \mathbb{R}^{M\times N}$ is the l^{th} spectral band of the datacube with $M \times N$ number of pixels, $\mathbf{C}_l \in \{0,1\}^{M\times N}$ is the Coded Aperture (CA) and $\mathbf{\Omega} \in \mathbb{R}^{M\times N}$ is the Gaussian noise.
The behavior of our MMSFA by 3D sphere packing with 3 MUX is represented as follows

CA and sphere for OSP-MSFA and 3 MUX



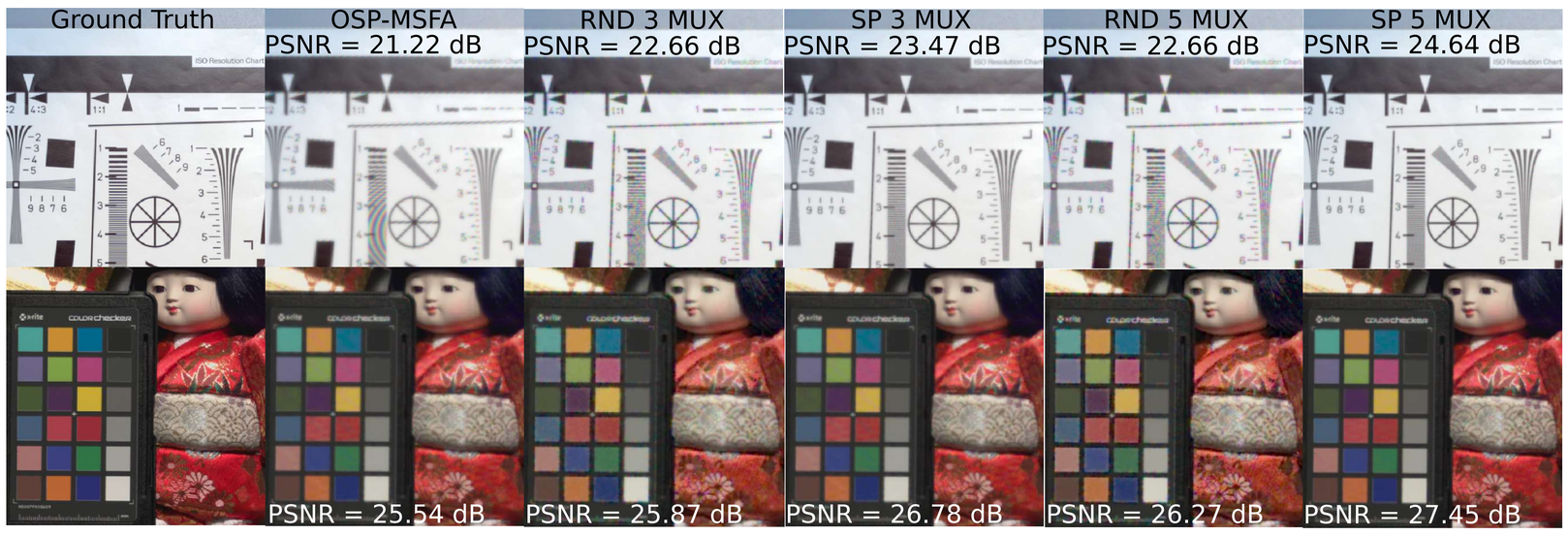
Sphere Packing Comparison

Sphere representation of OSP-MSFA[3] and MMSFA with 3 and 5 MUX



Simulations Results

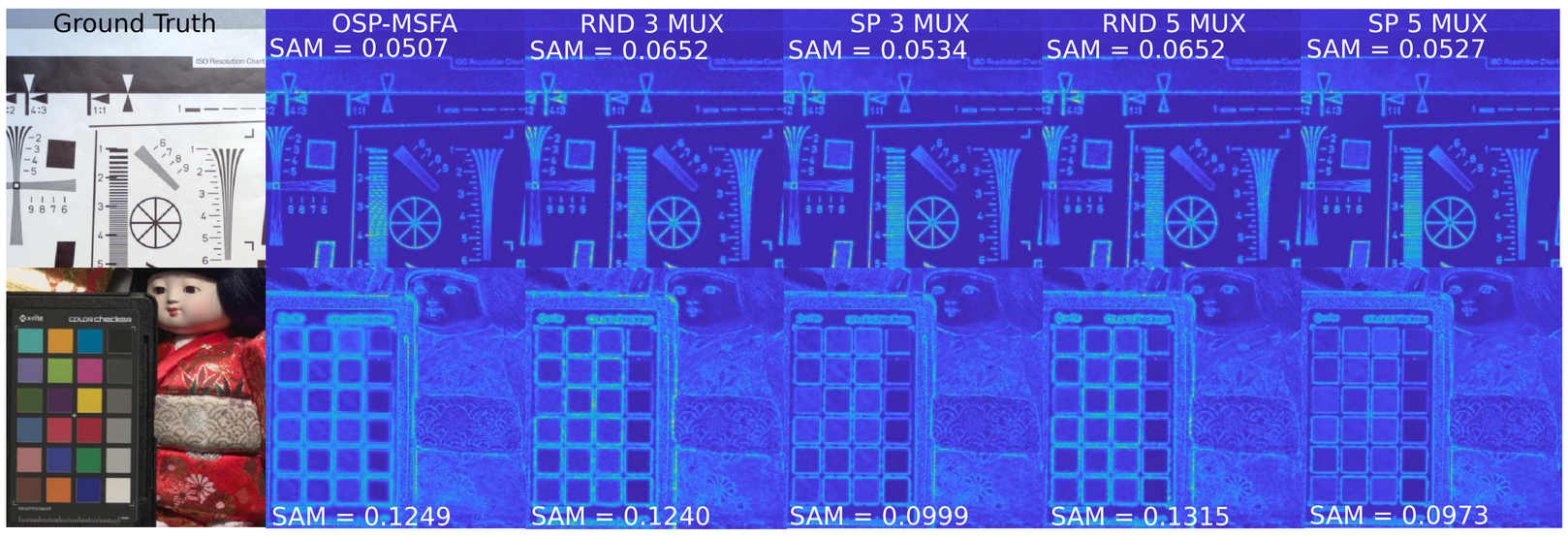
Spatial quality comparison from Tokyotech dataset with spectral resolution of 56 bands



Performance of MMSFA on the Tokyo dataset

Algorithms	Metrics	No Mux	RND 3 Mux	SP 3 Mux	RND 5 Mux	SP 5 Mux
PnP-GAP-3DTV	PSNR (dB) ↑	24.86	25.46	26.35	25.92	**27.13**
	SSIM ↑	0.7455	0.7804	0.8122	0.8025	**0.8300**
	SAM ↓	**0.0781**	0.0982	0.0845	0.1028	0.0837

Spectral quality comparison from Tokyotech dataset with spectral resolution of 56 bands



Bibliography

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[2] T. C. Hales, "A proof of the kepler conjecture," Annals of mathematics, vol. 162, no. 3, 2005.
[3] N. Diaz, A. Alvarado, P. Meza, F. Guzman, and E. Vera, "Multispectral filter array design by optimal sphere packing," IEEE TIP. 32, 2023.

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Conclusions

• The advantages of Sphere Packing approach include: extend the MSFA to a MMSFA, and reducing artifacts such as false colors and the zipper effect of reconstruction algorithms. Additionaly, this method extend the number of filters that would be acquired avoiding SNR reduction.