Compressive spectral video by optimal 4D-sphere packing

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Abstract: This work presents a mathematical framework to design 4D-coded aperture (CA)s for compressive snapshot spectral video (SV) exploiting 4D-sphere packing (SP). Simulation results using state-of-the-art datasets and metrics show a promising performance of the proposed approach to capture high-dimensional datacubes with limited sensing resources.

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1. Introduction

Sensing spectral video (SV) involves capturing a sequence of spectral datacubes over time, where each spectral datacube hyperspectral images (HI) has two spatial dimensions and one spectral dimension. SV may find outstanding applications in environmental sciences or medicine, tracking dynamic processes that have not been properly studied using traditional spectral imaging devices.

Nevertheless, capturing SV is a challenging problem in the 4D-Euclidean space, where a conventional approach to acquire the SV relies on scanning to complete the sequence of datacubes, giving up on temporal resolution. Since scanning is a time-consuming process, compressive sensing (CS) has emerged as a sensing protocol to capture a compressed version of the signal, where the underlying signal can be later recovered by solving an ill-posed inverse problem. Traditionally, the encoding patterns known as CA are random realizations. In contrast, novel approaches have shown outstanding performance using uniform patterns, and those approaches are known as SP-driven methods [1, 2], inspired by early methods based on Fourier analysis [3]. SP is a problem in discrete geometry that consists of packing congruent balls in a container such that only the spheres touch each other in a tangential manner [4]. In detail, optimal SP in 1D is trivial because the points are separated by a corresponding diameter whose resulting density is $\eta_1 \approx 1$, SP in 2D is circular packing with density $\eta_2 \approx 0.90$. 3D-SP or the Kepler conjecture packs 3D spheres, the resulting density is $\eta_3 \approx 0.74$. Recently, 3D-SP has been successfully applied to temporal sensing [1] and MSFA design [2]. Specifically, in SV the container is a hypercube with size $K \times K \times K \times K$ and the number of balls is $L = K^3$ because the number of spheres is proportional to the sensor's size and the number of frames. In this work, we introduce 4D-SP to design CA, and our approach exploits the best-known 4D-sphere packing density $\eta_4 = \frac{\pi^2}{16} = 0.61\ldots$ in the 4D-Euclidean space of the $\mathbf{D}_4 \in \mathbb{Z}^{4\times 4}$ lattice.

2. Methods

The corresponding compressive measurement corresponds to the following discrete model:

$$\mathbf{Y} = \sum_{t=0}^{T-1}\sum_{k=0}^{K-1} \mathcal{X}_{(:,:,k,t)} \odot \mathcal{C}_{(:,:,k,t)} + \mathbf{\Omega}, \tag{1}$$

where K is the number of bands and T is the number of frames, $\mathcal{X}_{(:,:,k,t)} \in \mathbb{R}^{M\times N}$ denotes the spectral temporal scene at the k^{th} band, and the t^{th} frame, the coded aperture is given by $\mathcal{C}_{(:,:,k,t)} \in \mathbb{R}^{M\times N}$, let $\odot$ be the Hadamard product, and $\mathbf{\Omega} \in \mathbb{R}^{M\times N}$ is the additive noise. In 4-dimensional Euclidean space the best-known packing density corresponds to the following lattice $\mathbf{D}_4 = [-1,-1,0,0;1,-1,0,0;0,1,-1,0;0,0,1,-1]$, and the corresponding Gram matrix is $\mathbf{A}_4 = \mathbf{D}_4^T\mathbf{D}_4$. The density of a 4D-lattice is given by

$$\frac{\text{Vol}(S_r^4)}{\text{Vol}(\mathbb{R}^4/\mathbf{D}_4)} = \frac{\frac{\pi^2}{2}r^4}{\sqrt{\det(\mathbf{A}_4)}} = 0.61\ldots \tag{2}$$

where $\text{Vol}(S_r^4) = \frac{\pi^2}{2}r^4$ is the volume of the 4−dimensional sphere and $\text{Vol}(\mathbb{R}^4/\mathbf{D}_4) = \sqrt{\det(\mathbf{A}_4)}$ is the volume of the lattice, the radius is $r = 1/\sqrt{2}$.

2.1. *Coding optimization strategy*

The sampling of SV can leverage the following solution to 3DN^2QP [5] to place the spheres within a 3D-container $\mathcal{B}$ as follows:

$$\mathcal{B}_{(:,:,t)} = ((\alpha \odot \mathbf{V} + \beta \odot \mathbf{H} + \gamma \odot q_t) \mod K + \mathbf{1}), \tag{3}$$