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Homework 5: Principal Component Analysis for Multispectral Compression

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The assignment report is due on 21 October, 2026. In it, you must solve the exercises presented in the guide in report format, and attach the files you generate, such as .py, ipynb, .mat, .m, .fig, etc. The report should briefly describe the experiments performed, including the following sections: summary, description of experiments, and conclusions. It is suggested that you prepare the report in LaTeX to facilitate writing equations. Send a compressed folder with the files to the Aula submission corresponding unit, with the subject line: **homework5_FotografiaComputacional** and the compressed folder's name **homework5_nameSurname.zip**, where Name is your first name and Surname is your last name. Caution: any attempt at copying or fraud will invalidate the lab and result in a zero on the lab grade.

Learning Goal

To implement the principal component transformation to reconstruct multispectral images.

1. Introduction

A spectral image is one that reproduces the form of an object based on the wavelength that the object in question is reflecting (or emitting); or, put another way, it is a set of images of the same object, each represented at a different wavelength, as can be seen in figure 1 and 2. The conventional visible multispectral images are in the range between 400 nm and 700 nm.

1.1. Principal Component Analysis (PCA)

In statistics, principal component analysis is a technique used to reduce the dimensionality of a dataset. Equations are used for this purpose

$$\mathbf{m}_x = \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k \quad (1)$$

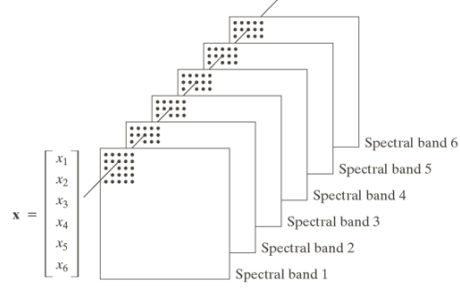


Figura 1: Tensor example of a multispectral image.

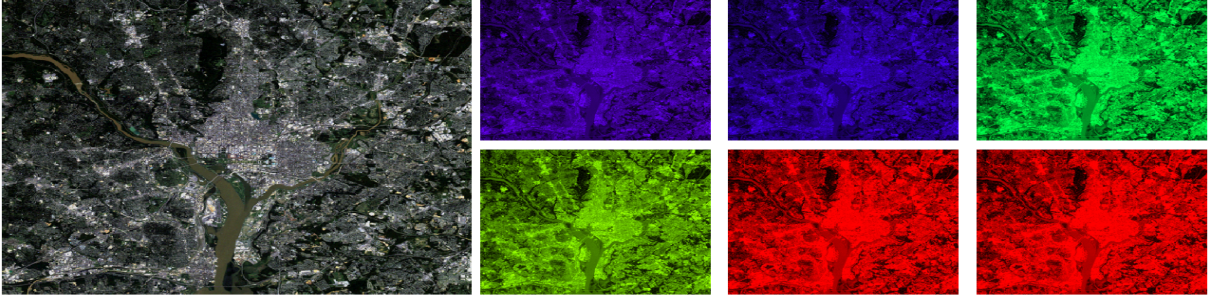


Figura 2: Multispectral image with six spectral bands.

and

$$\mathbf{C}_x = \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k \mathbf{x}_k^T - \mathbf{m}_k \mathbf{m}_k^T. \quad (2)$$

Let $\mathbf{A}$ be a matrix whose rows are formed by the eigenvectors of the matrix $\mathbf{C}_x$, ordered from largest to smallest according to their eigenvalues. Based on this, we have that

$$\mathbf{y} = \mathbf{A}(\mathbf{x} - \mathbf{m}_x). \quad (3)$$

Since the rows of matrix $\mathbf{A}$ are orthonormal vectors, it follows that $\mathbf{A}^{-1} = \mathbf{A}^T$, and that any vector $\mathbf{x}$ can be reconstructed from its corresponding measures $\mathbf{y}$, by using the expression

$$\mathbf{x} = \mathbf{A}^T \mathbf{y} + \mathbf{m}_x. \quad (4)$$

Using only the first k principal components, we obtain that

$$\hat{\mathbf{x}} = \mathbf{A}_k^T \mathbf{y} + \mathbf{m}_x. \quad (5)$$

One example of this equation is shown in Figure 3.

2. Exercises

- To find the mean vector $\mathbf{m}_x$ of the spectral signatures and its covariance matrix $\mathbf{C}_x$.
- To compute the eigenvector and eigenvalues of the covariance matrix $\mathbf{C}_x$
- To reconstruct the image using only its first two principal components.
- To select 10 images from the Tokio dataset, perform PCA compression and determine how many principal components are necessary to recover the data with 90 % accuracy in terms of average PSNR, SSIM and SAM.

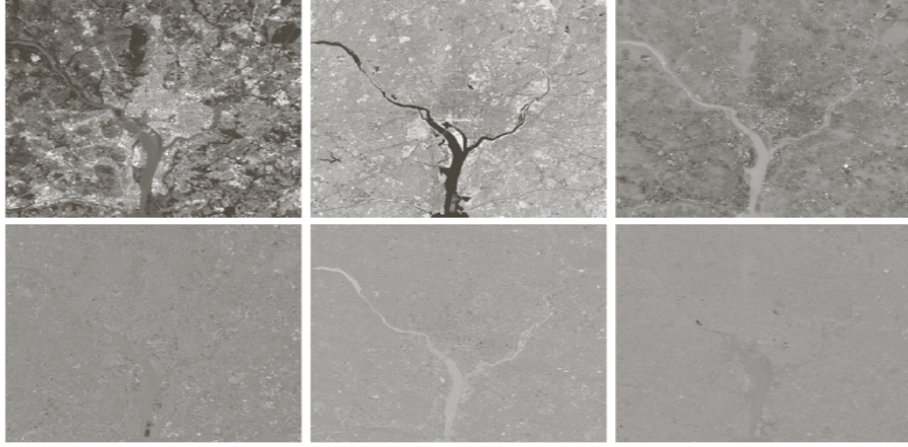


Figura 3: Example of an image reconstructed from six principal eigenvalues.

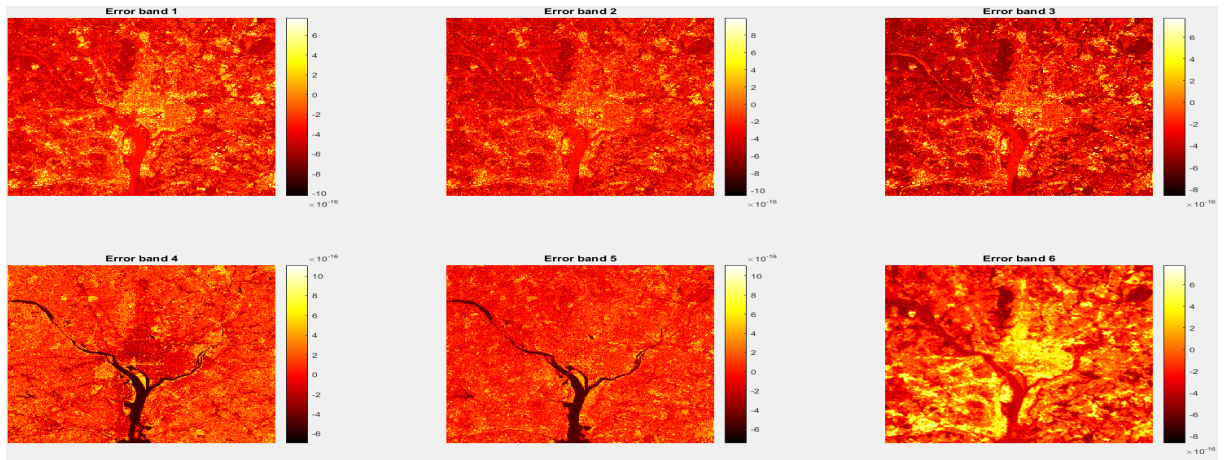


Figura 4: Example of the absolute error of the reconstructed image in Figure 2, based on its six principal eigenvalues.